

Quantum Physics II (8.05) Fall 2004

Practice Problems 13

Massachusetts Institute of Technology
Physics Department
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Nothing to be handed in!

This week we study the quantum mechanics of more than one particle, Fermi-Dirac and Bose-Einstein statistics, and the construction of states with proper symmetry under particle exchange. The related problems in combining angular momentum will also be treated.

Reading Assignment for week 13 of the course

- Griffiths §5.1, on Identical Particles
- For additional reading on identical particles see Ohanian §9.4, Cohen-Tannoudji, §XIV, and Shankar §10.

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Practice problems for this week

1. Bosons and Fermions in a Square Well

Consider a set of 4 identical particles confined in a one-dimensional infinitely high square well of length L .

- (a) What are the single particle energy levels? What are the corresponding single particle wave functions? Name the wave functions $\phi_1(x)$, $\phi_2(x)$. . .
- (b) Suppose the particles are spinless bosons. What is the energy and (properly normalized) wave function of the ground state? Of the first excited state? Of the second excited state? [Example: The ground state is $\Psi(x_1, x_2, x_3, x_4) = \phi_1(x_1)\phi_1(x_2)\phi_1(x_3)\phi_1(x_4)$.]
- (c) If the particles are spin 1/2 fermions, what is the energy and wave function of the ground state? The first excited state? The second excited state?

Hint: use some abbreviated notation such as a Slater determinant. For example, the ground state is

$$\Psi(x_1, x_2, x_3, x_4) = \frac{1}{\sqrt{4!}} \begin{vmatrix} \phi_1^\uparrow(x_1) & \phi_1^\downarrow(x_1) & \phi_2^\uparrow(x_1) & \phi_2^\downarrow(x_1) \\ \phi_1^\uparrow(x_2) & \phi_1^\downarrow(x_2) & \phi_2^\uparrow(x_2) & \phi_2^\downarrow(x_2) \\ \phi_1^\uparrow(x_3) & \phi_1^\downarrow(x_3) & \phi_2^\uparrow(x_3) & \phi_2^\downarrow(x_3) \\ \phi_1^\uparrow(x_4) & \phi_1^\downarrow(x_4) & \phi_2^\uparrow(x_4) & \phi_2^\downarrow(x_4) \end{vmatrix} \quad (1)$$

with energy

$$E = \frac{\hbar^2 \pi^2}{2mL^2} (1 + 1 + 4 + 4) .$$

2. Griffiths 2nd edition, Problem 5.4 (or in the 1st edition, 5.3)

3. Griffiths 2nd edition, Problem 5.6 (or in the 1st edition, 5.5)

4. Two Electrons

Consider two electrons. Their spatial wave function may be either symmetric or antisymmetric under the interchange of the electrons' coordinates. Since the electrons are fermions, the overall wave function must be antisymmetric under the simultaneous interchange of both space coordinates and spin.

- (a) Suppose the spatial wave function is antisymmetric. What are the allowed spin wave functions of the electrons? What are the eigenvalues of the square and the z -component of the total spin operator $\vec{S} = \vec{s}_1 + \vec{s}_2$ in these states?
- (b) Repeat part (a) in the case where the space wave function is symmetric.

- (c) Suppose that (up to this point in the problem) all the states enumerated in parts (a) and (b) have the same energy. Now add the following term to the Hamiltonian:

$$H' = C\vec{s}_1 \cdot \vec{s}_2 .$$

[That is, the electrons interact by a spin-spin force due to the interaction of the magnetic moment of each with the magnetic field generated by the other.] What are the eigenstates of the system including the interaction H' ? What are the energies of the states in parts (a) and (b)?

5. Two Electrons in a Spin Singlet State

Consider two electrons in a spin-singlet state.

- (a) If a measurement of the spin of one of the electrons shows that it is in an S_z -eigenstate with S_z -eigenvalue $\hbar/2$, what is the probability that a measurement of the z -component of the spin of the other electron yields $\hbar/2$?
- (b) If a measurement of the spin of one of the electrons shows that it is in an S_y -eigenstate with S_y -eigenvalue $\hbar/2$, what is the probability that a measurement of the x -component of the spin of the other electron yields $-\hbar/2$?