

$$\text{Hooke: } \epsilon = \epsilon_0 \left| \sin \frac{qa}{2} \right|, \quad \phi(\epsilon) = \begin{cases} \frac{2N_i}{\pi\epsilon_0} \frac{1}{\sqrt{1 - (\frac{\epsilon}{\epsilon_0})^2}}, & 0 < \epsilon < \epsilon_0 \\ 0, & \epsilon \leq 0, \epsilon \geq \epsilon_0 \end{cases} \quad U_i = \frac{2N_i\epsilon_0}{\pi} \int_0^1 \frac{x}{\sqrt{1-x^2}} \left(\frac{1}{e^{\beta\epsilon_0 x} - 1} + \frac{1}{2} \right) dx$$

$$\text{Debye: } \epsilon = \begin{cases} \frac{\epsilon_0 a}{2} |q|, & |q| \leq \frac{\pi}{a} \\ 0, & |q| > \frac{\pi}{a} \end{cases} \quad \phi(\epsilon) = \begin{cases} \frac{2N_i}{\pi\epsilon_0}, & 0 < \epsilon < \frac{\pi}{2}\epsilon_0 \\ 0, & \epsilon \leq 0, \epsilon \geq \frac{\pi}{2}\epsilon_0 \end{cases} \quad U_i = \frac{\pi N_i \epsilon_0}{2} \int_0^1 x \left(\frac{1}{e^{\frac{\pi}{2}\beta\epsilon_0 x} - 1} + \frac{1}{2} \right) dx$$

$$\text{Einstein: } \epsilon = \epsilon_0, \quad \phi(\epsilon) = N_i \delta(\epsilon - \epsilon_0), \quad U_i = N_i \epsilon_0 \left(\frac{1}{e^{\beta\epsilon_0} - 1} + \frac{1}{2} \right)$$

