Electron in a quantum well

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When an electron moves in a fixed potential area, then there is no sense of force only at the boundary of the region. In nature, the potential is a function normally smooth that can be approached by a constant space potential. Starting from the one-dimensional Schrodinger' s equation

$$\Psi''(x) + \left(\frac{2m}{\hbar^2}\right)\left(E - V(x)\right)\Psi(x) = 0 \ (1)$$

we calculate the eigenfunctions and the eigenvalues. Here, we study the finite potential well.

$$V(x) = \begin{cases} 0, & -\frac{a}{2} < x < \frac{a}{2} \\ V_0, & x < -\frac{a}{2}, x > \frac{a}{2} \end{cases}$$

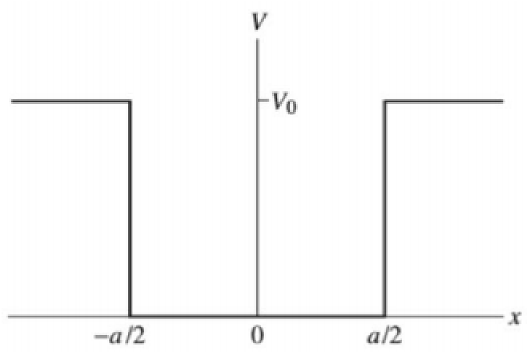

Figure 1 : Finite potential well (https://www.chegg.com/study)

The solution of Schrodinger's equation

We solve Schrodinger's equation in three different regions. We use the following equations to solve the problem:

I. $\lim_{x\to-\infty} \Psi(x) = 0 \, , \lim_{x\to\infty} \Psi(x) = 0 \ (2)$

II. $\Psi_L(x) = \Psi_R(x) \, , \Psi'_L(x) = \Psi'_R(x) \ (3)$

Equations (3) are called matching conditions. Due to the symmetry of the potential, the wave functions are even and odd functions.

- Even solutions $\Psi(x) = \Psi(-x)$: $\Psi(x) = \begin{cases} Ae^{k_1 x}, x < -\frac{a}{2} \\ \tilde{D}\cos k_2 x, -\frac{a}{2} < x < \frac{a}{2} \\ Ae^{-k_1 x}, x > \frac{a}{2} \end{cases}$

- <u>Odd solutions $\Psi(x) = -\Psi(-x)$</u> : $\Psi(x) = \begin{cases} Ae^{k_1 x}, x < -\frac{a}{2} \\ \tilde{C}cosk_2 x, -\frac{a}{2} < x < \frac{a}{2} \\ -Ae^{-k_1 x}, x > \frac{a}{2} \end{cases}$

$$k_1^2 = \frac{2m}{\hbar^2}(V_0 - E)\,, k_2^2 = -\frac{2m}{\hbar^2}$$

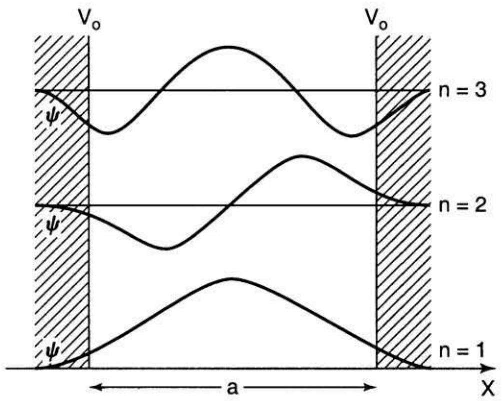

Figure 2: The ground state and the two first excited states. The probability to find the electron out of the well is not zero. The even and the odd wave functions are interchanged. (http://what-when-how.com/)

As opposed to infinite potential well, the wave function in the regions 1 and 3 is not zero. Out of the well the wave function is in the form of an exponential wave but in the well the wave function has a trigonometric form. The eigenvalues of energy can't be calculated by a closed analytical form, as in the infinite potential well.

References

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